

Non-Perturbative Effects of Geometry in Wide-Angle Redshift Distortions

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21 February 2008

ABSTRACT

We use the formalism of Szapudi (2004) to extend the theory of large angle redshift distortions. We identify a previously neglected term in the Jacobian, which contains a purely geometric non-linearity while still being linear in terms of the dynamics. We perform linear perturbation theory while keeping these non-perturbative terms intact. The results represent a significant correction to previous calculations and are in excellent agreement with our measurements in the Hubble volume simulation. It appears that the new formulae amount to a satisfactory description of redshift distortions on large scales.

1 INTRODUCTION

Redshift distortions represent a curse disguised as a blessing for high precision cosmological applications. Radial coordinates of redshift surveys contain limited phase space information, which in principle can be used to constrain theories more than configuration information alone; moreover, velocities are sensitive to structure outside of the survey boundaries which potentially translates into a larger “effective volume”. On the other hand, redshift distortions are plagued with non-linearities, both on large and small scales, therefore in worst case they could amount to poorly understood contamination of the configuration space data. Our aim is to extend the theory of linear redshift distortions such that large angle information could be successfully extracted from galaxy surveys.

The work of Davis & Peebles (1983) and Peebles (1980) showing that redshift distortions affect the power spectrum spawned a lot of activity. The all-important linear, plane-parallel limit was first calculated by Kaiser (1987), showing that the effect on the power spectrum corresponds to “squashing”. The other well known “fingers of God” effect dominates small scales, and is irrelevant for our present study. The Kaiser formula has been generalized for real space soon after (e.g., Hamilton 1993; Cole et al. 1995). These theories have been used to analyze surveys such as the Point Source Catalog Redshift (PSCz) Survey (Tadros et al. 1999), the Two-Degree Field Galaxy Redshift Survey (2dFGRS; Peacock et al. 2001; Hawkins et al. 2003; Tegmark et al. 2002), and the Sloan Digital Sky Survey (SDSS; Zehavi et al. 2002).

The distant observer approximation only holds if pairs are separated by a small angle. This means that a large fraction of pairs needs to be thrown away from modern wide angle redshift surveys when they are analyzed in this limit. These pairs are typically fewer and noisier than close pairs,

but if our aim is to extract as much information as possible from a given survey, it would be desirable to add them in. This motivated Szalay et al. (1998) to perform a full linear perturbation theory calculation without the distant observer limit. The calculation in this results a finite expression only in coordinate space: in Fourier space an infinite series will result for the redshift distorted analog of the power spectrum. These formulae were later further generalized to include high- z effects in various cosmologies by Matsubara et al. (2004). These results provided the theoretical foundation of several subsequent analyses of wide angle redshift surveys, such as Pope et al. (2004) and Okumura et al. (2007). Despite its elegance, the theory did not agree well with dark matter simulations. Scoccimarro (2004) pointed out that this might be due to non-perturbative effects.

Szapudi (2004) reanalyzed the redshift distortion problem from group theoretical point of view showing that tripolar spherical harmonics provide an excellent basis for expansion, and result in especially compact formulae. In addition it provided specific coordinate systems, one of which recovers the Legendre expansion of Szalay et al. (1998), while the other represents the same information in an even simpler Fourier mode expansion. We use this formalism to take into account another term in the Jacobian neglected by all previous work. This term, while linear in terms of the small fluctuations, is essentially non-linear from the point of view of geometry: it contains a $1/r$ prefactor, which was also the justification for neglecting it. Moreover, this term, if expanded in terms of bipolar spherical harmonics (or any other way), would contribute infinite terms. In this paper we introduce a hybrid approach, where we leave the essentially non-perturbative terms in the expansion, i.e. our tripolar expansion coefficients will still contain angular variables. As we show later, this procedure results in a finite number of terms, and it provides dramatic improvement in the

agreement with simulations. In retrospect, the omission of this term, while intuitively reasonable, is not justified, as its contribution can be critical on the most interesting scales of tens of $h^{-1}\text{Mpc}$'s.

In the next §2 we present the theory of linear redshift distortions including results from the newly identified geometric term in the Jacobian. We follow closely the formalism of Szapudi (2004), mainly focusing on the new aspects of this calculation. For reference, we print the full result, which has about twice as many terms as previously. In §3 we compare our results with preliminary measurements in the Hubble volume simulations, and present our conclusions.

2 REDSHIFT DISTORTION OF THE TWO-POINT CORRELATION FUNCTION

We use linear perturbation theory to predict the redshift distorted two-point correlation function in terms of the underlying power spectrum. Our calculation is based directly on the tripolar expansion formalism of Szapudi (2004), therefore our focus will be on the additional terms arising from the Jacobian.

The exact mapping between real and redshift space is $s_i = x_i - f v_j \hat{x}_i \hat{x}_j$, where the "hat" denotes the proper unit vector, $f = \frac{\Omega^{0.6}}{b}$ and the velocity has units which provides that its divergence is equal to the density up to linear order. From this, one can calculate the derivative of this matrix: $\partial s_i / \partial x_k = \delta_{ik} + O_{ik}(v)$ where O is linear in v . This results in a linear Jacobian $J = 1 + \text{Tr} O = 1 - f \hat{x}_i \hat{x}_j \partial_i v_j - 2f \frac{x_j v_j}{x^2}$. The last term in the previous expression is usually omitted due to the fact that it scales with $1/x$, i.e. it would tend to zero for large distances, which loosely correspond to large angles as well. Closer examination of this term shows that it is of the same order as the previous term, not only in perturbation expansion (linear), but also in order of magnitude. Our goal is to propagate this new term through the full calculation.

The linear density contrast and the two-point function can be expressed in the usual fashion.

$$\delta_s(x) = \int \frac{d^3 k}{(2\pi)^3} e^{ik_j x_j} \left[1 + f(\hat{x}_j \hat{k}_j)^2 - i2f \frac{\hat{x}_j \hat{k}_j}{xk} \right] \delta(k) \quad (1)$$

$$\begin{aligned} \langle \delta_s(x_1) \delta_s^*(x_2) \rangle &= \int \frac{d^3 k}{(2\pi)^3} P(k) e^{ik(x_1 - x_2)} \\ &\quad \left[1 + \frac{f}{3} + \frac{2f}{3} P_2(\hat{x}_1 \hat{k}) - \frac{i2f}{x_1 k} P_1(\hat{x}_1 \hat{k}) \right] \\ &\quad \left[1 + \frac{f}{3} + \frac{2f}{3} P_2(\hat{x}_2 \hat{k}) + \frac{i2f}{x_2 k} P_1(\hat{x}_2 \hat{k}) \right], \quad (2) \end{aligned}$$

where P_1 and P_2 are Legendre polynomials and $P(k)$ is the linear power spectrum. The third term in each of the brackets correspond to the extension of the previous results; these would tend to zero in the plane parallel limit. At wide angles, the separation between the galaxies and the distance between a galaxy and the observer are of the same order, therefore kx is of order unity. This shows explicitly that the order of this term can be as large as the previous, and the detailed calculation confirms this.

Next we express the angular dependence of the correlation function with tripolar spherical harmonics.

$$S_{l_1 l_2 l}(\hat{x}_1, \hat{x}_2, \hat{x})$$

$$\equiv \sum_{m_1, m_2, m} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix} C_{l_1 m_1}(\hat{x}_1) C_{l_2 m_2}(\hat{x}_2) C_{lm}(\hat{x}) \quad (3)$$

We use x for denoting $x_1 - x_2$. On the right hand side one can find the Wigner $3j$ symbols and we define the normalized spherical functions as $C_{lm} = \sqrt{4\pi/2l+1} Y_{lm}$; these latter result in simpler expressions.

Eq. (2) has become more complex with the additions, x_1 and x_2 appear in the denominator resulting in the following angular dependence

$$x_1 = g_1 x = \frac{\sin(\phi_2)}{\sin(\phi_2 - \phi_1)} x \quad (4)$$

$$x_2 = g_2 x = \frac{\sin(\phi_1)}{\sin(\phi_2 - \phi_1)} x. \quad (5)$$

Expanding these terms into tripolar spherical harmonics would yield infinite terms, but simplification arises from the fact that they can be factored out of the integrals. All the rest can be expanded as in Szapudi (2004), resulting in finite expressions. We introduce ϕ_1 to denote the angle between \hat{x}_1 and \hat{x} and ϕ_2 for the angle between \hat{x}_2 and \hat{x} . We emphasize that the coefficients of this (quasi-)tripolar expansion still has an angular dependence in the form of g_1 and g_2 :

$$\xi_s = \sum_{l_1 l_2 l} B^{l_1 l_2 l}(x, \phi_1, \phi_2) S_{l_1 l_2 l}(\hat{x}_1, \hat{x}_2, \hat{x}). \quad (6)$$

After performing the expansions, only a finite number of coefficients survive. For reference, the ones from Szapudi (2004) are:

$$\begin{aligned} B^{000}(x) &= (1 + \frac{1}{3}f)^2 \xi_0^2(x) \\ B^{220}(x) &= \frac{4}{9\sqrt{5}} f^2 \xi_0^2(x) \\ B^{022}(x) = B^{202}(x) &= -(\frac{2}{3}f + \frac{2}{9}f^2) \sqrt{5} \xi_2^2(x) \\ B^{222}(x) &= \frac{4\sqrt{10}}{9\sqrt{7}} f^2 \xi_2^2(x) \\ B^{224}(x) &= \frac{4\sqrt{2}}{\sqrt{35}} f^2 \xi_4^2(x); \quad (7) \end{aligned}$$

and the new terms, the main result of this paper, are

$$\begin{aligned} B^{101}(x, \phi_1, \phi_2) &= -(2f + \frac{2}{3}f^2) \frac{\sqrt{3}}{g_1 x} \xi_1^1(x) \\ B^{011}(x, \phi_1, \phi_2) &= (2f + \frac{2}{3}f^2) \frac{\sqrt{3}}{g_2 x} \xi_1^1(x) \\ B^{121}(x, \phi_1, \phi_2) &= \frac{4\sqrt{2}}{\sqrt{15}} f^2 \frac{1}{g_1 x} \xi_1^1(x) \\ B^{211}(x, \phi_1, \phi_2) &= -\frac{4\sqrt{2}}{\sqrt{15}} f^2 \frac{1}{g_2 x} \xi_1^1(x) \\ B^{123}(x, \phi_1, \phi_2) &= \frac{4\sqrt{7}f^2}{\sqrt{15}g_1 x} \xi_3^1(x) \\ B^{213}(x, \phi_1, \phi_2) &= -\frac{4\sqrt{7}f^2}{\sqrt{15}g_2 x} \xi_3^1(x) \\ B^{110}(x, \phi_1, \phi_2) &= -\frac{4f^2}{\sqrt{3}g_1 g_2 x^2} \xi_0^0(x) \\ B^{112}(x, \phi_1, \phi_2) &= -\frac{4\sqrt{10}f^2}{\sqrt{3}g_1 g_2 x^2} \xi_2^0(x), \quad (8) \end{aligned}$$

where $\xi_l^m(x) = \int dk / 2\pi^2 k^m j_l(xk) P(k)$ with j being the spherical Bessel function.

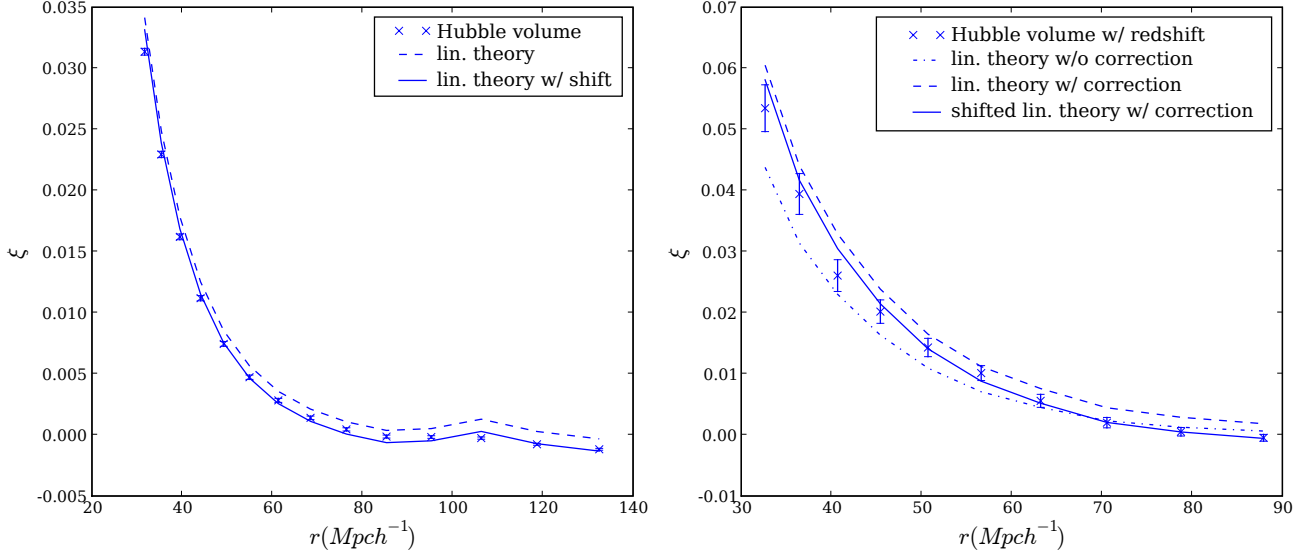


Figure 1. (Left) The measurement of the correlation function without redshift distortion of the Hubble volume simulation (dots) compared with linear theory (dashed and solid lines). The errorbars were estimated from 9^3 subvolumes of the Hubble volume. Shifting the theory by 0.001 downward, motivated by the integral constraint, provides an excellent fit to the data. (Right) Redshift distorted correlation function of the Hubble volume simulation (dots) at constant opening angle (0.71 radian) and while the ratio of the distances of the particles in the pair are kept fixed (at 1.57). The errorbars were estimated as before. The lines indicate the linear theories with (this paper) and without the geometric terms. The solid line is the corrected theory with a downshift of 0.0024. The integral constraint correction is expected to be larger since the average of the two point function is larger.

For further elaboration we choose coordinate system a) from Szapudi (2004). This corresponds to our previous choice of angles with ϕ_1, ϕ_2 , with which the $S_{l_1 l_2 l}(\hat{x}_1, \hat{x}_2, \hat{x}) = S_{l_1 l_2 l}(\pi/2, \phi_1, \pi/2, \phi_2, \pi/2, 0)$ functions can be expressed using cosines and sines only. Using the same notation as Szapudi (2004):

$$\xi_s(\phi_1, \phi_2, x) = \sum_{n_1, n_2=0,1,2} a_{n_1 n_2} \cos(n_1 \phi_1) \cos(n_2 \phi_2) + b_{n_1 n_2} \sin(n_1 \phi_1) \sin(n_2 \phi_2). \quad (9)$$

Again, for reference, the previously calculated coefficients are

$$\begin{aligned} a_{00} &= \left(1 + \frac{2f}{3} + \frac{2f^2}{15}\right) \xi_0^2(x) - \left(\frac{f}{3} + \frac{2f^2}{21}\right) \xi_2^2(x) + \frac{3f^2}{140} \xi_4^2(x) \\ a_{02} = a_{20} &= \left(\frac{-f}{2} - \frac{3f^2}{14}\right) \xi_2^2(x) + \frac{f^2}{28} \xi_4^2(x) \\ a_{22} &= \frac{f^2}{15} \xi_0^2(x) - \frac{f^2}{21} \xi_2^2(x) + \frac{19f^2}{140} \xi_4^2(x) \\ b_{22} &= \frac{f^2}{15} \xi_0^2(x) - \frac{f^2}{21} \xi_2^2(x) - \frac{4f^2}{35} \xi_4^2(x); \end{aligned} \quad (10)$$

and the new expressions of this work correspond to

$$\begin{aligned} a_{10} &= \frac{\tilde{a}_{10}}{g_1} = (2f + \frac{4f^2}{5}) \frac{1}{g_1 x} \xi_1^1 - \frac{1}{5} \frac{f^2}{g_1 x} \xi_3^1 \\ a_{01} &= \frac{\tilde{a}_{01}}{g_2} = -(2f + \frac{4f^2}{5}) \frac{1}{g_2 x} \xi_1^1 + \frac{1}{5} \frac{f^2}{g_2 x} \xi_3^1 \\ a_{11} &= \frac{\tilde{a}_{11}}{g_1 g_2} = \frac{4}{3} \frac{f^2}{g_1 g_2 x^2} \xi_0^0 - \frac{8}{3} \frac{f^2}{g_1 g_2 x^2} \xi_2^0 \end{aligned}$$

$$\begin{aligned} a_{21} &= \frac{\tilde{a}_{21}}{g_2} = -\frac{2}{5} \frac{f^2}{g_2 x} \xi_1^1 + \frac{3}{5} \frac{f^2}{g_2 x} \xi_3^1 \\ a_{12} &= \frac{\tilde{a}_{12}}{g_1} = \frac{2}{5} \frac{f^2}{g_1 x} \xi_1^1 - \frac{3}{5} \frac{f^2}{g_1 x} \xi_3^1 \\ b_{11} &= \frac{\tilde{b}_{11}}{g_1 g_2} = \frac{4}{3} \frac{f^2}{g_1 g_2 x^2} \xi_0^0 + \frac{4}{3} \frac{f^2}{g_1 g_2 x^2} \xi_2^0 \\ b_{21} &= \frac{\tilde{b}_{21}}{g_2} = -\frac{2}{5} \frac{f^2}{g_2 x} \xi_1^1 - \frac{2}{5} \frac{f^2}{g_2 x} \xi_3^1 \\ b_{12} &= \frac{\tilde{b}_{12}}{g_1} = \frac{2}{5} \frac{f^2}{g_1 x} \xi_1^1 + \frac{2}{5} \frac{f^2}{g_1 x} \xi_3^1. \end{aligned} \quad (11)$$

It is worth to emphasize again that the angular dependence g_1 and g_2 is suppressed for clarity in the above formulae, but it is obviously carries through according to the definition of these functions. If the equivalence of the configurations $(\phi_1, \phi_2) \rightarrow (\pi - \phi_2, \pi - \phi_1)$ is taken into account (same pairs can be counted twice), the number of independent new coefficients is five, i.e. the number of terms approximately doubled. Next we explore the relevance of these calculations, and compare the theoretical predictions with measurements in dark matter only N -body simulations.

3 DISCUSSION AND SUMMARY

To understand our results, we expanded our formulae to identify leading order corrections to the Kaiser limit.

The leading order corrections to the distant observer approximation are second order. Using the notation $\frac{1}{2}(\phi_1 + \phi_2) = \phi$ and $\frac{1}{2}(\phi_2 - \phi_1) = \Delta\phi$, and keeping leading order terms in $\Delta\phi$ results in

$$\xi_s(\phi, \Delta\phi, x)$$

$$\begin{aligned}
&= a_{00} + 2a_{02} \cos(2\phi) + a_{22} \cos^2(2\phi) + b_{22} \sin^2(2\phi) \\
&\quad + \left[-4a_{02} \cos(2\phi) - 4a_{22} - 4b_{22} \right] \Delta\phi^2 \\
&\quad + \left[-4\tilde{a}_{10} \cot^2(\phi) + 4\tilde{a}_{11} \cot^2(\phi) \right. \\
&\quad \left. - 4\tilde{a}_{12} \cot^2(\phi) \cos(2\phi) + 4\tilde{b}_{11} - 8\tilde{b}_{12} \cos^2(\phi) \right] \Delta\phi^2 + \\
&\quad + O(\Delta\phi^4). \quad (12)
\end{aligned}$$

The first line of eq. (12) corresponds to the Kaiser formula ($\Delta\alpha = 0$). The next line contains leading order corrections corresponding to previous work only, and the third line collects leading order corrections from the new term in the Jacobian. These are all of the same order, reassuring the need of keeping the geometric non-perturbative terms. We conjecture that the terms containing the $\cot^2(\phi)$ could be responsible for the reported failure of the linear theory for small angles along the line of sight (Okumura et al. 2007).

As a preliminary test of the validity of our calculations, we measured correlation functions in the Hubble volume simulation (Evrard et al. 2002), using cosmological parameters $\sigma_8 = 0.9$, $n_s = 1$, $\Omega_m = 0.3$, $\Omega_\lambda = 0.7$, $h = 0.7$, $\Omega_b h^2 = 0.0196$ and a volume of $(3000 \text{ Mpc} h^{-1})^3$, with and without redshift distortions. The volume of the simulation was divided into 9^3 subvolumes to obtain the errorbars.

The left panel of the figure shows the measured and the theoretical two-point functions without redshift distortions. The theory agrees with the measurements only after a shift by a constant. This could be due to the “integral constraint” problem (Peebles 1980, e.g.), possibly compounded with slight non-linear effects.

Next, an observer was placed at the center of each sub-volume and the mapping between real and redshift space was performed using the velocities recorded in the simulation. The correlation function was then measured using brute force counting of pairs in high resolution bins matching our choice of coordinate system described earlier. The right panel of the figure presents wide angle redshift distortion theory both with and without non-perturbative geometric corrections. The theory without corrections cannot be made to agree with the measurements even using a constant offset. In contrast, the theory presented in this paper provides excellent agreement with the measurements if the effects of integral constraint are taken into account. Note that the shift corresponding to the latter is expected to be larger with redshift distortions included simply because the two-point function is enhanced on large scales.

While these measurements are preliminary in the sense that we did not try to span the full parameter space of wide angle redshift distortions, the results presented in this figure are typical in the sense that other configurations we measured showed similar improvement. Scanning the full parameter space with our present brute force two-point correlation function code would be impractical, since we need a very large number of pairs in each bin to beat down the errorbars enough that the difference between the two theories can be reliably measured. Although we developed a fast grid based code as well, we found that at these small values of the correlation function the pixel window function effects become important. These are more complex for the redshift distorted correlation function depending on three variables

than in real space. Such effect should be modeled very accurately before one could fully span the available parameter space. Such detailed modeling together with any other real-world issues affecting cosmological parameter estimation is left for subsequent research.

With these caveats we conclude that our theory of wide angle redshift distortions agrees with simulations, and that it represents an improvement over previous works, at modest cost in complexity. Possible generalizations along the lines of Matsubara et al. (2004) are left for future research.

4 ACKNOWLEDGMENT

We thank Mark Neyrinck for carefully reading the manuscript. This work was supported by NASA grant NNG06GE71G and NSF grant AMS04-0434413.

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